III. $\sin 0^\circ = 0$; $\cos 0^\circ = 1$; $\tan 0^\circ = 0$.

$\sin 30^\circ = \frac{1}{2}$; $\cos 30^\circ = \frac{\sqrt{3}}{2}$; $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

$\sin 45^\circ = \frac{1}{\sqrt{2}}$; $\cos 45^\circ = \frac{1}{\sqrt{2}}$; $\tan 45^\circ = 1$.

$\sin 60^\circ = \frac{\sqrt{3}}{2}$; $\cos 60^\circ = \frac{1}{2}$; $\tan 60^\circ = \sqrt{3}$.

$\sin 90^\circ = 1$; $\cos 90^\circ = 0$; $\tan 90^\circ = \infty$.

V. $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$; $\tan(-\theta) = -\tan\theta$.

$\sin(90^\circ - \theta) = \cos\theta$; $\sin(90^\circ + \theta) = \cos\theta$.

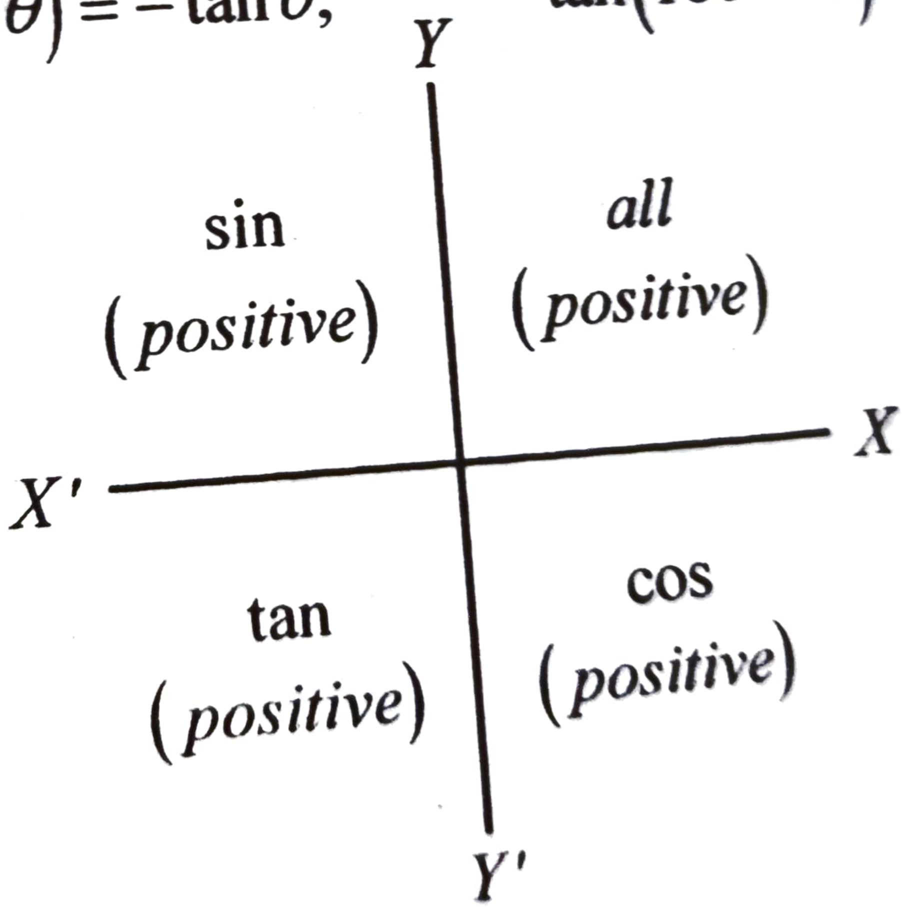
$\cos(90^\circ - \theta) = \sin\theta$; $\cos(90^\circ + \theta) = -\sin\theta$.

$\tan(90^\circ - \theta) = \cot\theta$; $\tan(90^\circ + \theta) = -\cot\theta$.

$\sin(180^\circ - \theta) = \sin\theta$; $\sin(180^\circ + \theta) = -\sin\theta$.

$\cos(180^\circ - \theta) = -\cos\theta$; $\cos(180^\circ + \theta) = -\cos\theta$.

$\tan(180^\circ - \theta) = -\tan\theta$; $\tan(180^\circ + \theta) = \tan\theta$.



... $[-1, 1] \to [-\pi/2, \pi/2]$ is shown in Fig. 4.3.

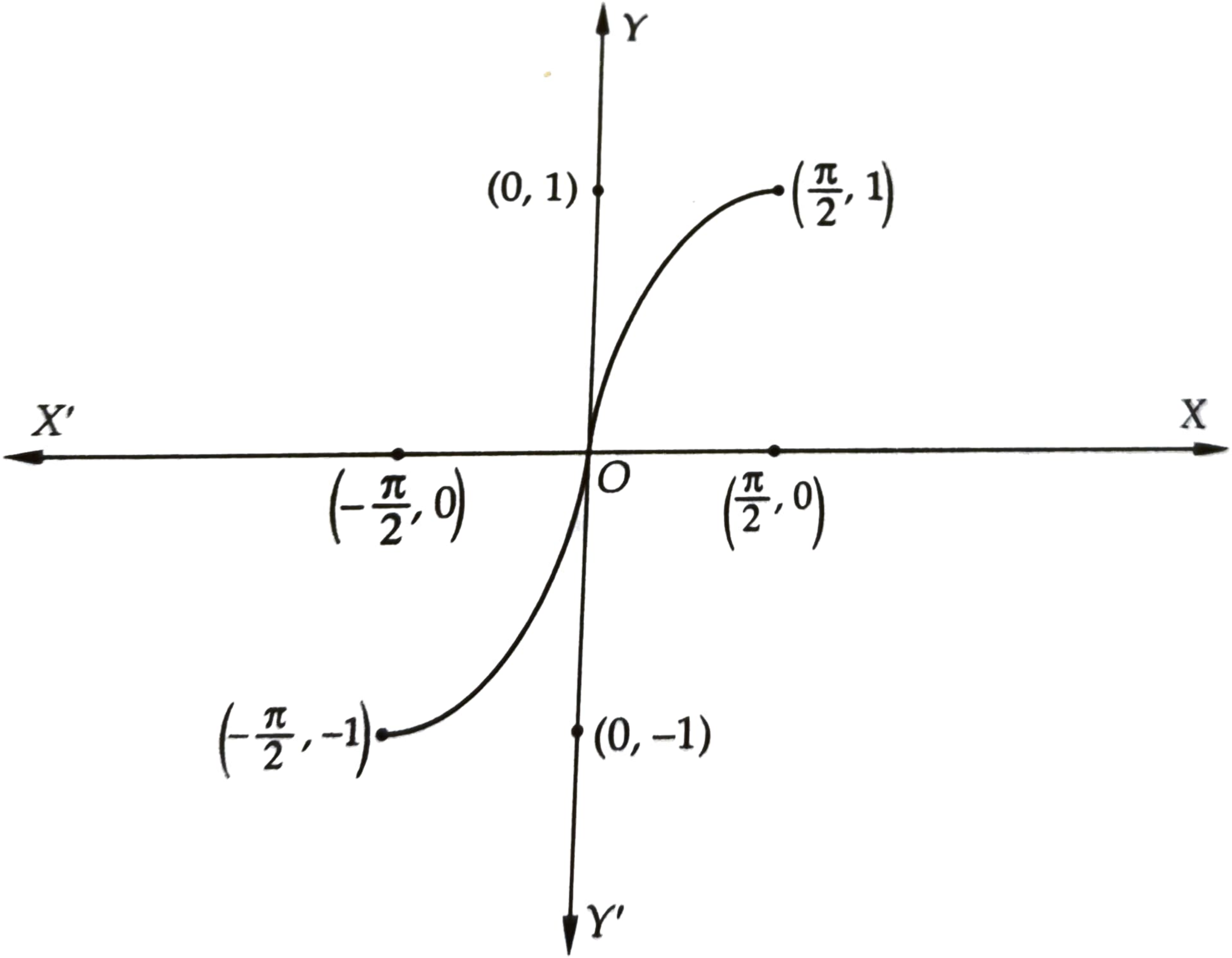


Fig.4.2 Graphof $y = \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

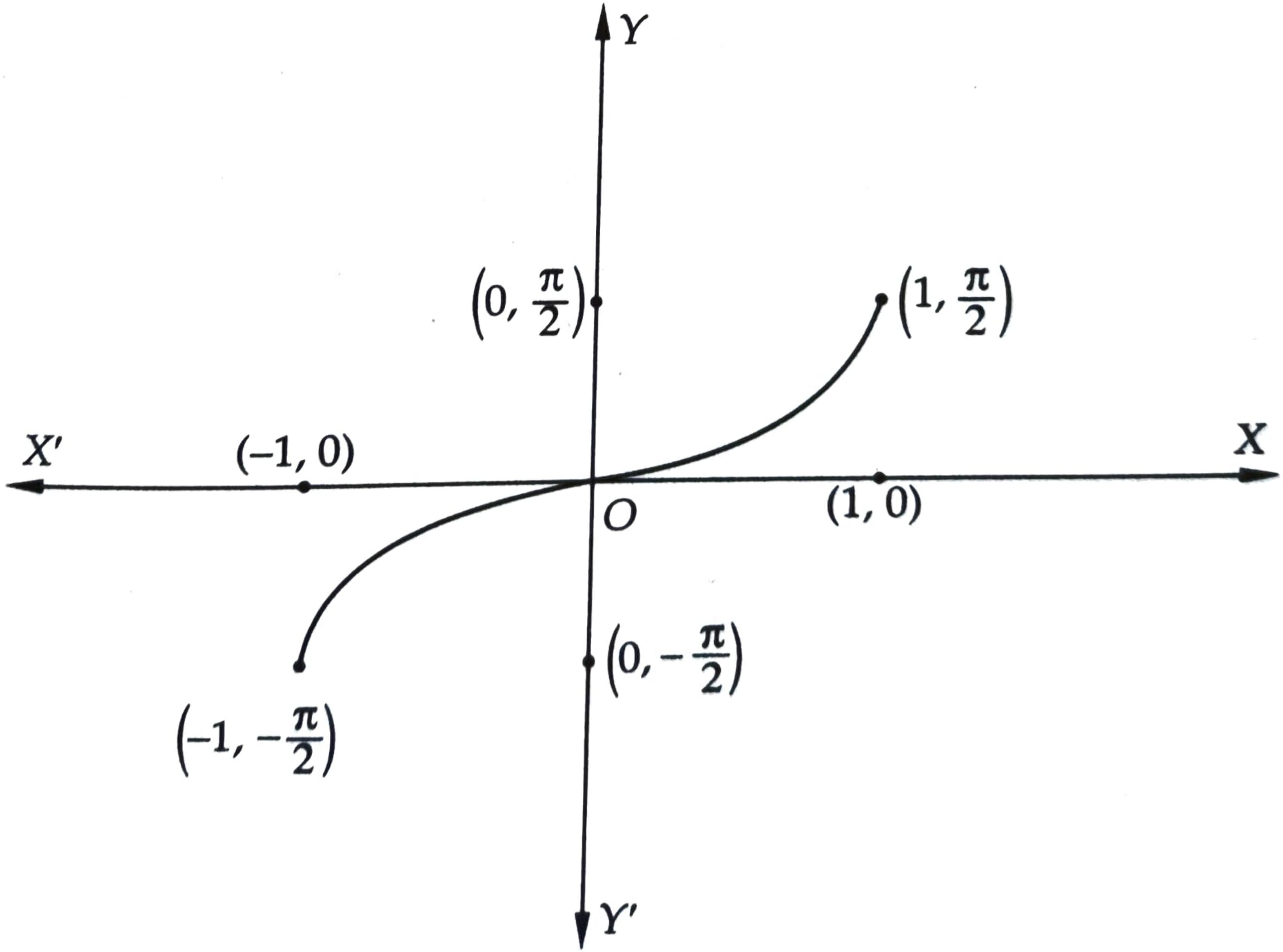


Fig. 4.3 Graph of $y = \sin^{-1} x, -1 \le x \le 1$

value of $\cos^{-1} x$ lying in $[0,$

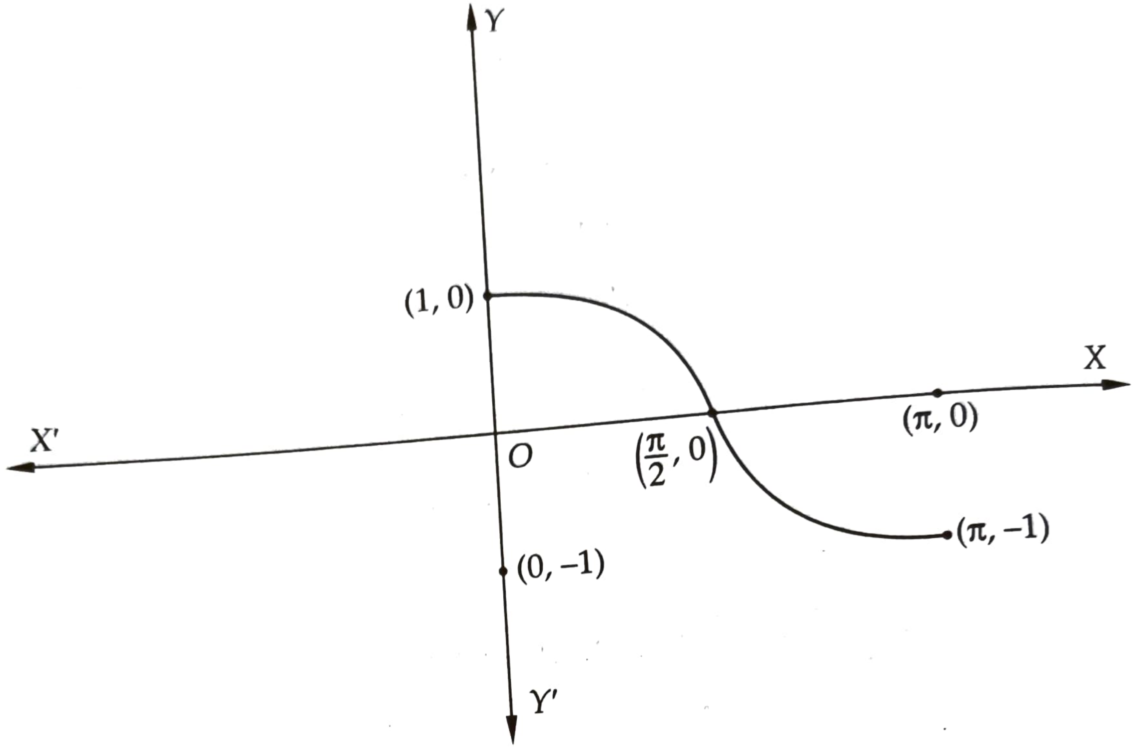


Fig. 4.7 Graph of $y = \cos x$, $0 \leq x \leq \pi$

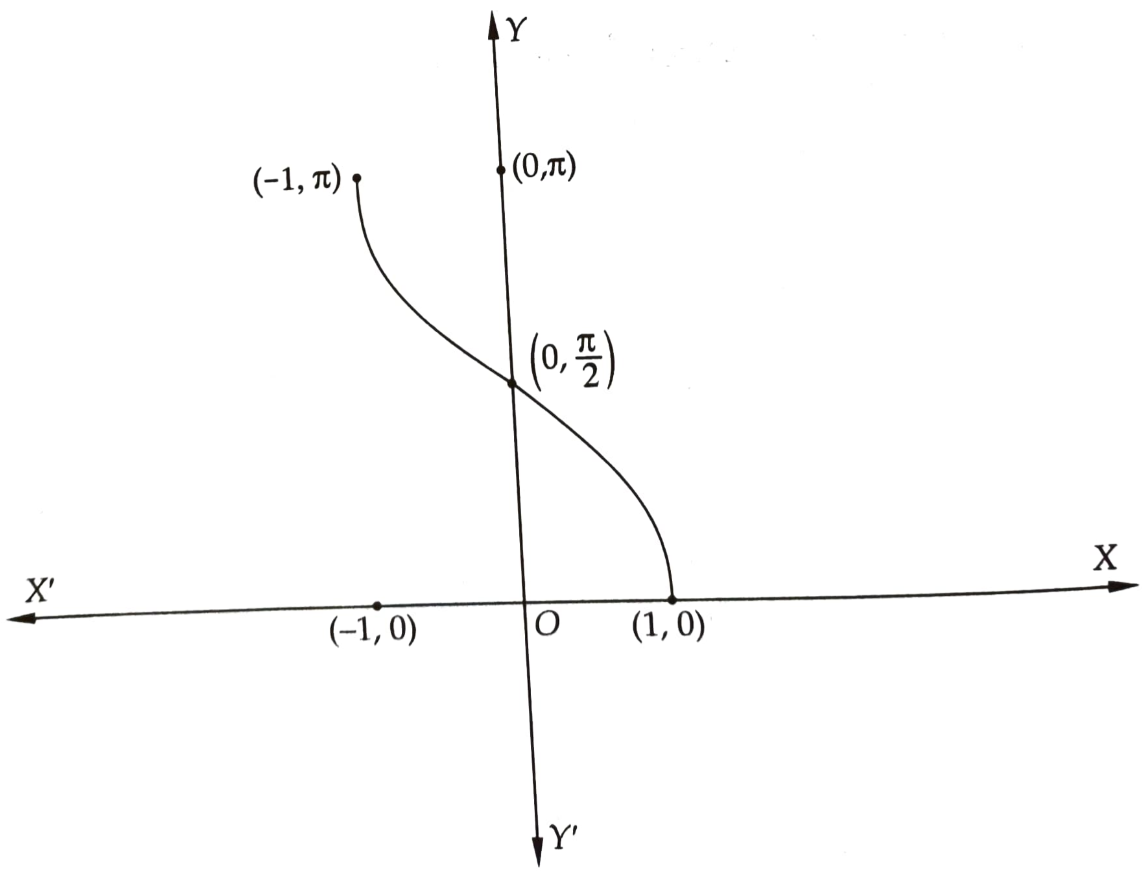


Fig. 4.8 Graph of $y = \cos^{-1} x$

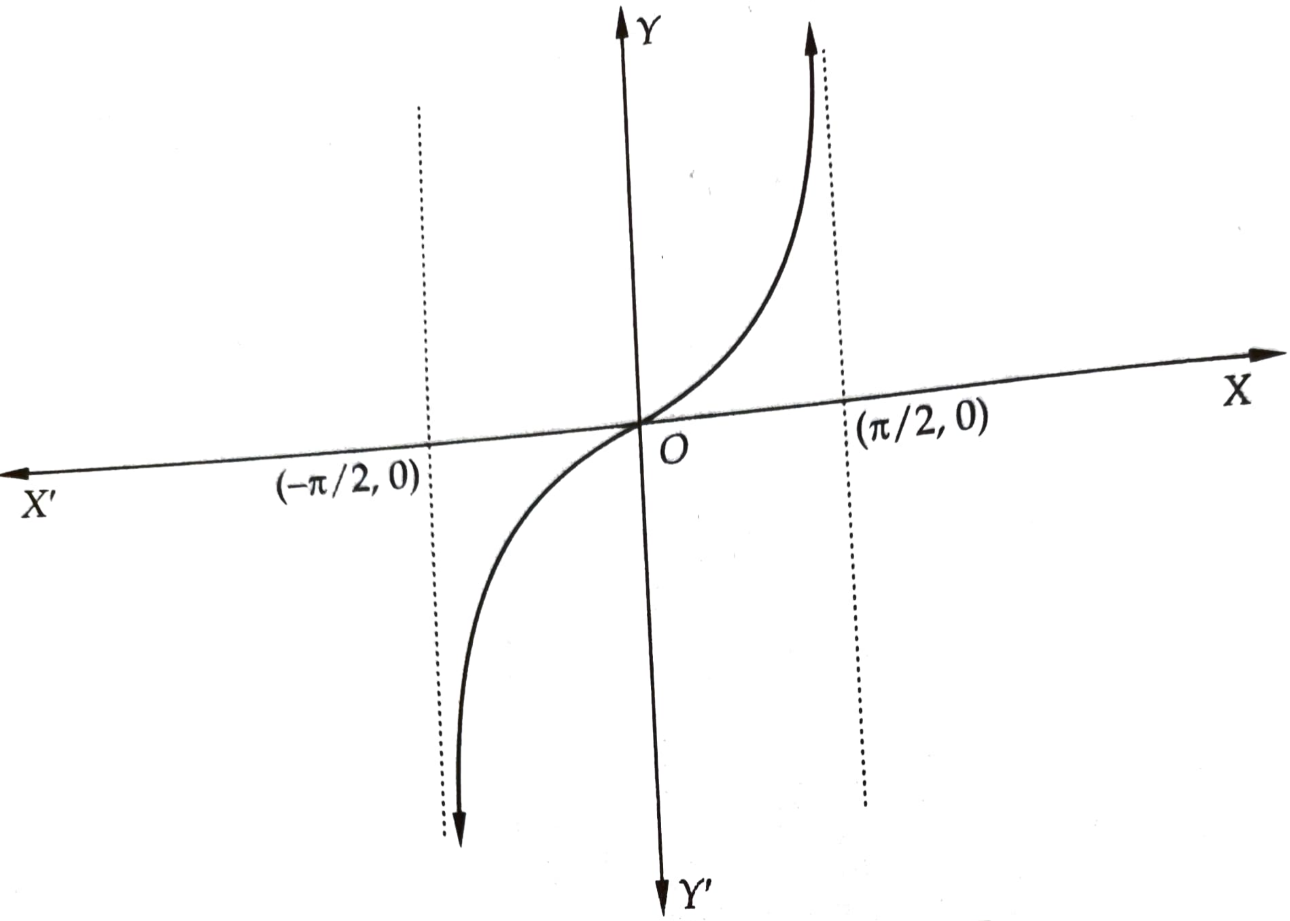


Fig. 4.10 Graph of $y = \tan x, -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

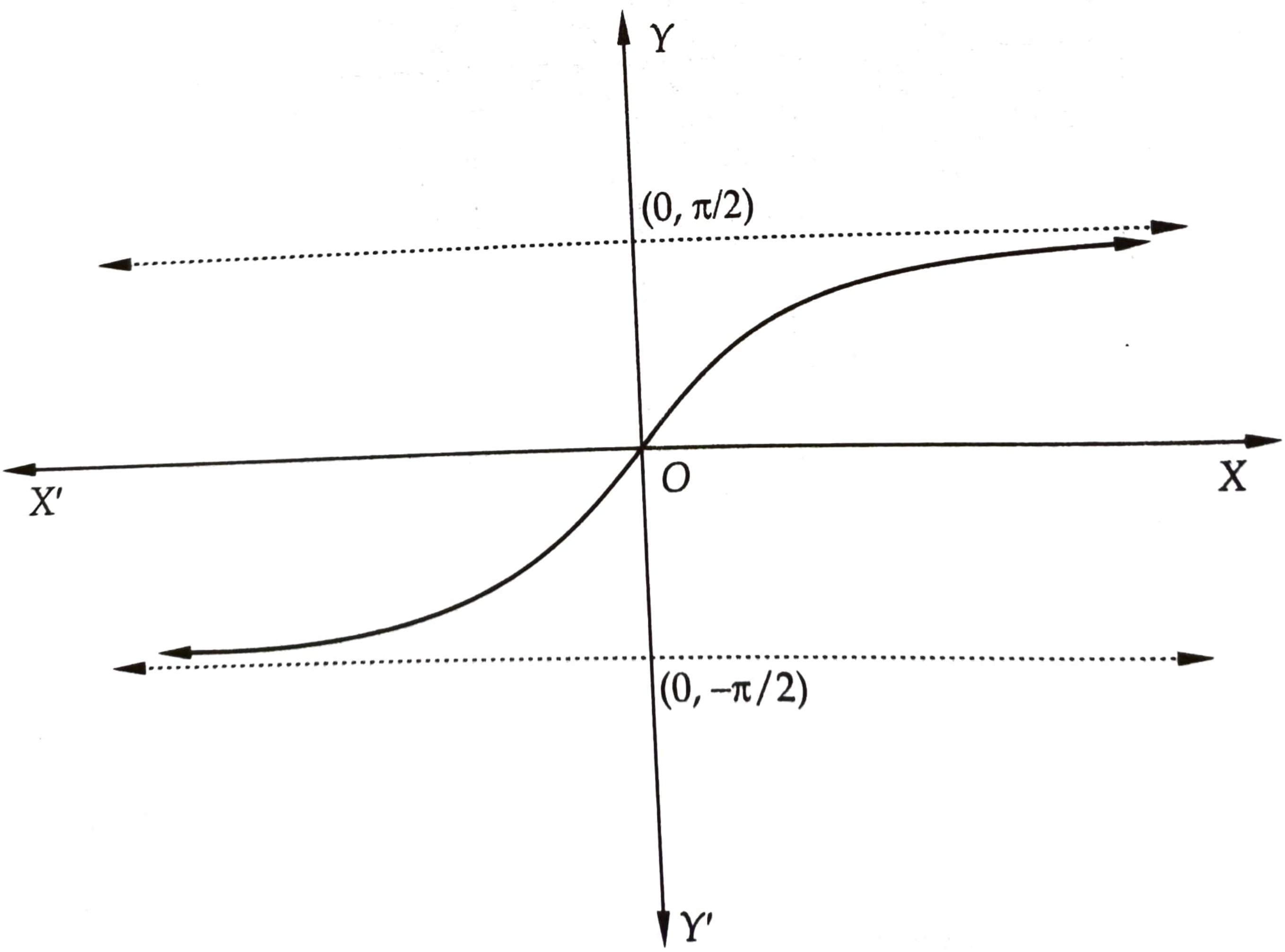


Fig. 4.11 Graph of $y = \tan^{-1} x$

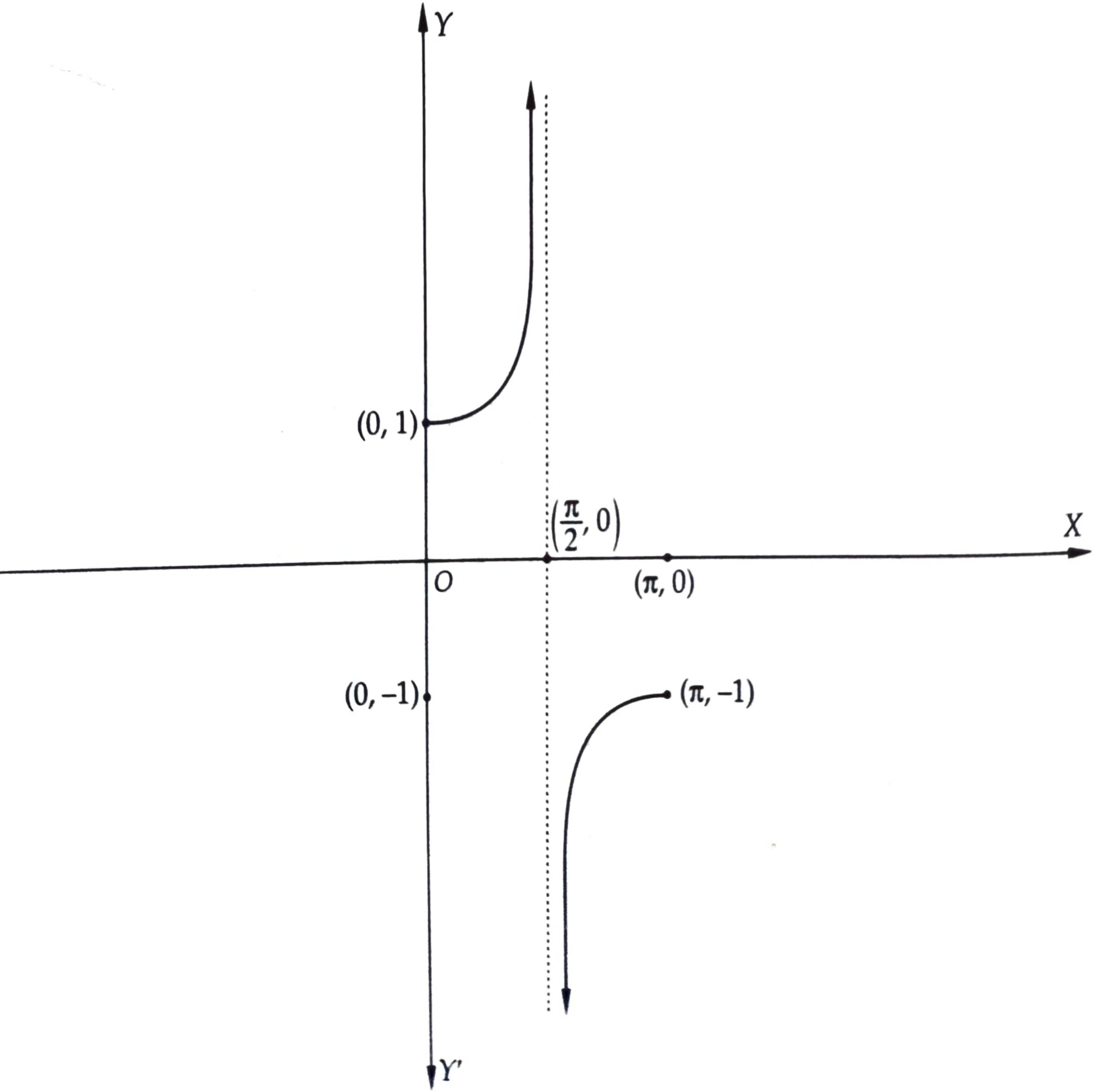


Fig. 4.13 Graph of $y = \sec x, 0 \leq x \leq \pi, x \neq \pi/2$

$\sec^{-1} x$ in $[0, \pi/2) \cup (\pi/2, \pi]$ for given value of $x \in ($

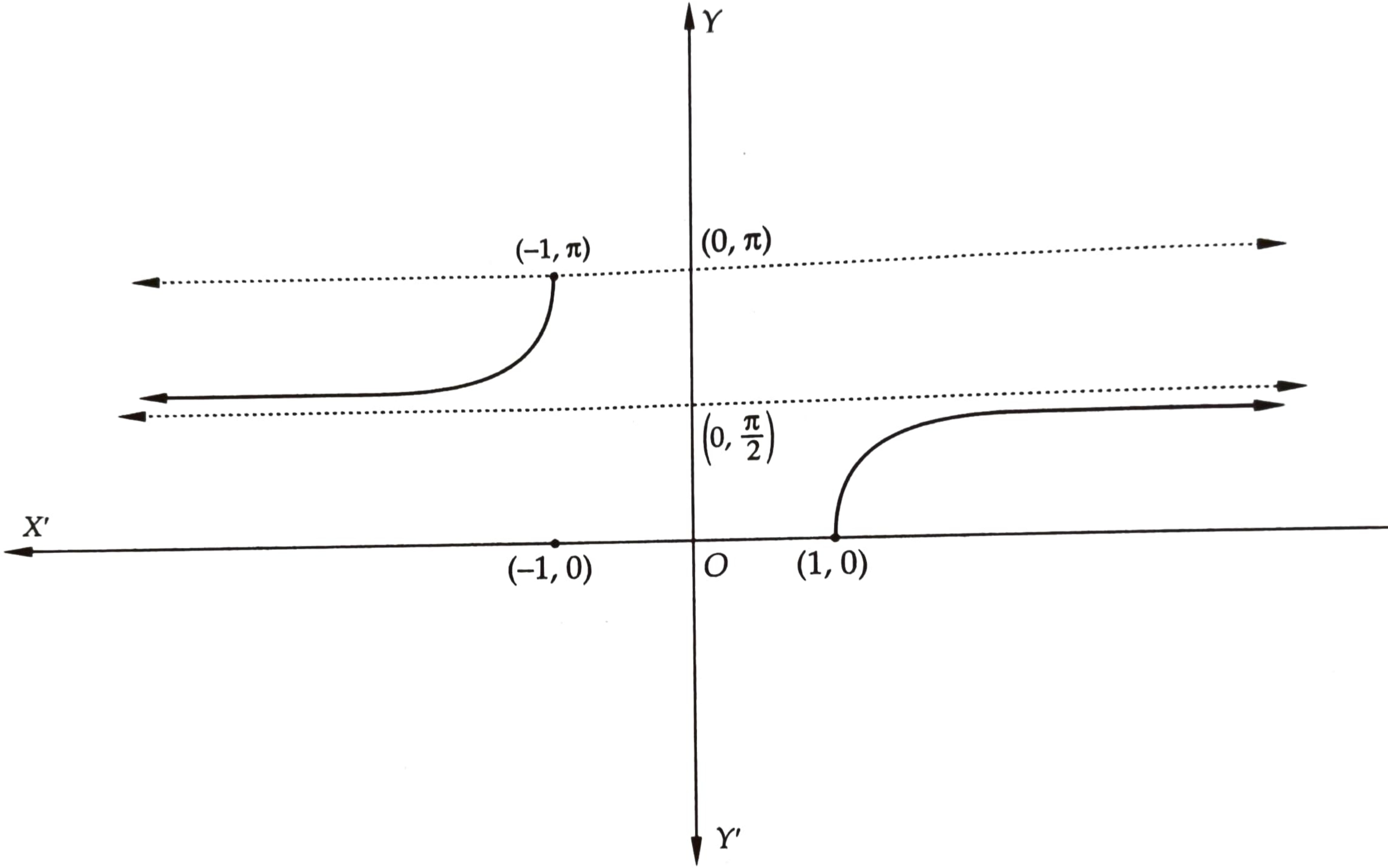


Fig. 4.14 Graph of $y = \sec^{-1} x$

$\to (-\infty, -1] \cup [1, \infty)$

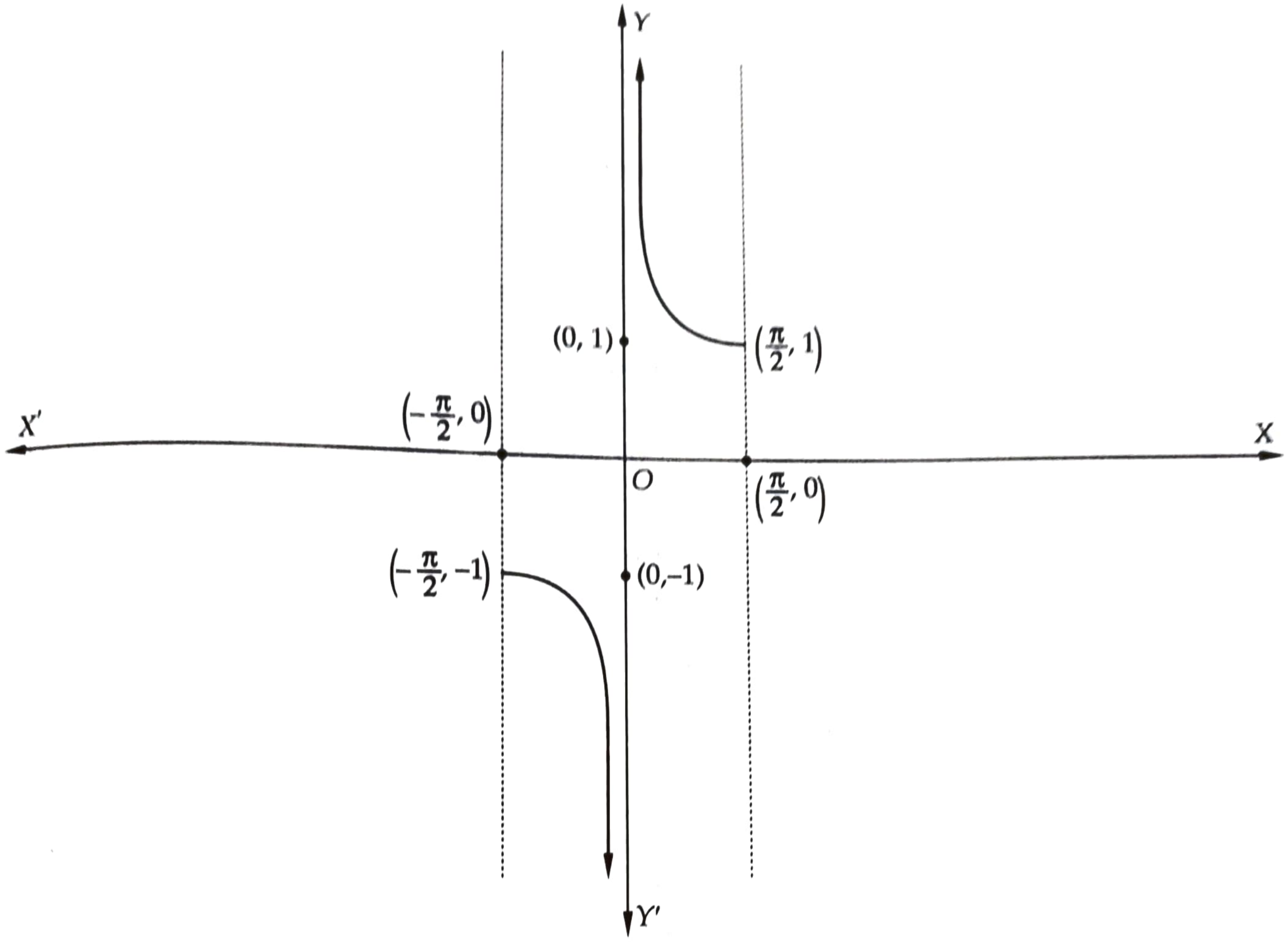


Fig. 4.16 Graph of $y = \operatorname{cosec} x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, x \neq 0$

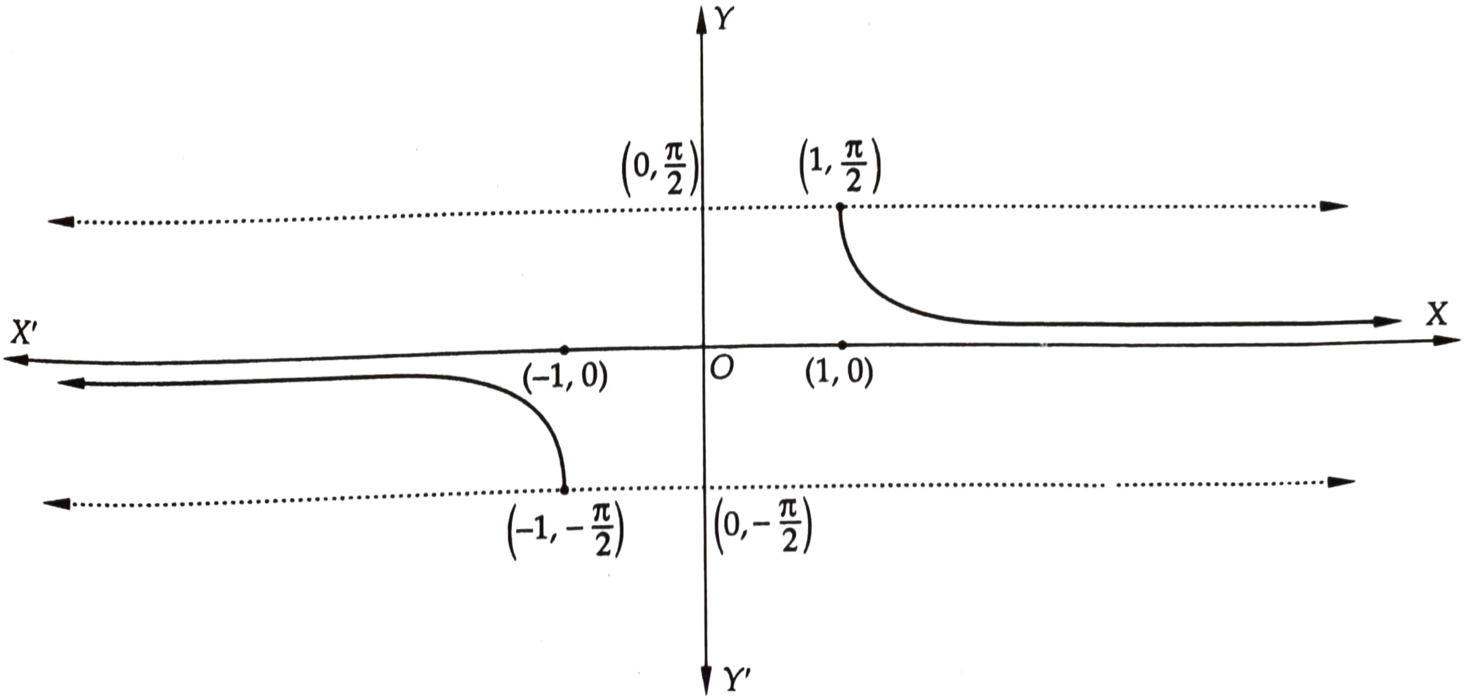


Fig. 4.17 Graph of $y = \operatorname{cosec}^{-1} x$

বিপৰীত বৃত্তীয় ফলন · INVERSE CIRCULAR Fn.

বিপৰীত ফলনৰ ~~আগতে~~ বাবে ফলন এটা একৈকী আৰু আচ্ছাদক হ'ব লাগে। sine, cos, tan, cot sec, cosec আদি ~~ছ~~ 6 টা trigonometric ফলন আছে। ফলনকেইটা 1-1 আৰু onto নহয় বাবে ইহঁতৰ বিপৰীত ফলন নাথাকে। কিন্তু আদিক্ষেত্ৰ আৰু সহক্ষেত্ৰক সীমিত কৰি আমি ইহঁতক 1-1 আৰু onto বনাব পাৰো।

(1) Sine ফলনৰ বিপৰীত ফলন :—

ধৰা $f : R \to R$ ফলন যাতে $f(x) = \sin x$.

আদিক্ষেত্ৰ $[-\frac{\pi}{2}, \frac{\pi}{2}]$ আৰু সহক্ষেত্ৰ $[-1, 1]$ লোলে $f(\theta) = \sin\theta$ ফলনটো এটা 1-1 আৰু onto ফলন হ'ব।

f^{-1} ক আমি $\sin^{-1}$ হিচাপে লিখো। গতিকে $\sin^{-1}$ ফলনৰ আদিক্ষেত্ৰ $[-1, 1]$ আৰু সহক্ষেত্ৰ $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

আমি জানো যে $\sin^{-1}x = \theta \Leftrightarrow \sin\theta = x$.

Note: আদিক্ষেত্ৰ $[-\frac{\pi}{2}, \frac{\pi}{2}]$ নলৈ $[-\frac{3\pi}{2}, -\frac{\pi}{2}]$, $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $[\frac{\pi}{2}, \frac{3\pi}{2}]$ আদি ল'ব পৰা যায়; সহক্ষেত্ৰ $[-1, 1]$.

$y = \sin x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ আৰু $y = \sin^{-1}x$ $-1 \le x \le 1$ ৰ চিত্ৰ photo কৰি দিয়া হ'ল। (মবাইলতো চাওক)

<u>Ex</u> নিচের সমান রাশিটির - মুখ্যমান (principal) নির্ণয় কর

(i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (ii) $\sin^{-1}\left(\frac{1}{2}\right)$

<u>Soln</u> ধরো $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$

$= \sin\frac{\pi}{3}$.

$\therefore$ ~~sin~~ $\Rightarrow x = \frac{\pi}{3}$

$\therefore \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ $\quad\left(-\frac{\pi}{2} < \frac{\pi}{3} < \frac{\pi}{2}\right)$

(ii) $\sin^{-1}\left(\frac{1}{2}\right) = x$

$\Rightarrow \sin x = \frac{1}{2} = \sin\frac{\pi}{6}$.

$\therefore x = \frac{\pi}{6}$.

$\Rightarrow \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Ans. $\quad \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Ex1 $f(x) = \sin^{-1}(2x-3)$ ফলনৰ domain উলিওৱা।

Sol^n $\sin^{-1}x$ ৰ আদিক্ষেত্ৰ $[-1,1]$.

$\therefore$ $f(x) = \sin^{-1}(2x-3)$ ফলনৰ ক্ষেত্ৰত

$-1 \le 2x-3 \le 1 \Rightarrow 3-1 \le 2x \le 3+1 \Rightarrow 1 \le x \le 2 \Rightarrow x \in [1$

Hence $f(x)$ ৰ আদিক্ষেত্ৰ হ'ব $[1,2]$

Ex 2 $f(x) = \sin^{-1}(-x^2)$ ৰ আদিক্ষেত্ৰ উলিওৱা।

Sol^n ফলনটো সংজ্ঞাবদ্ধ হ'বৰ বাবে x ৰ মান হ'ব

$-1 \le x^2 \le 1 \Rightarrow 1 \ge x^2 \ge -1 \Rightarrow 0 \le x^2 \le 1$ ($\because x^2$, ঋণাত্মক হ'ব নোৱাৰে)

$\Rightarrow x^2 \le 1 \Rightarrow x^2-1 \le 0 \Rightarrow (x-1)(x+1) \le 0 \Rightarrow -1 \le x \le 1$

$\therefore$ ফলন $f(x)$ৰ আদিক্ষেত্ৰ $[-1,1]$.

Ex 3. ~~$f(x) = \sin^{-1}x + \cos x$ ৰ domain উলিওৱা~~

Ex 3. $f(x) = \sin^{-1}\sqrt{x-1}$

ইয়াত $-1 \le \sqrt{x-1} \le 1$

$\Rightarrow 0 \le \sqrt{x-1} \le 1$

$\Rightarrow 0 \le x-1 \le 1 \Rightarrow 1 \le x \le 2 \Rightarrow x \in [1,2]$

cosine ফলনৰ বিপৰীত ফলন: Inverse of cosine F^n.

$f: R \to R$, $f(\theta) = \cos\theta$ ফলনটো 1-1 আৰু onto নহয়।

কিন্তু $f: ~~[0,]~~ [0, \pi] \to [-1, 1]$ ত ই বিপৰীত ফলন বিশিষ্ট।

$\therefore f^{-1}: [-1, 1] \to [0, \pi]$.

Q. $\cos^{-1}(2x-1)$ৰ আদিক্ষেত্ৰ উলিওৱা

Soln $\cos^{-1}x$ ৰ আদিক্ষেত্ৰ $[-1, 1]$.

এতিয়া $-1 \leq 2x-1 \leq 1$

$\Rightarrow 0 \leq 2x \leq 2$

$\Rightarrow 0 \leq x \leq 1$

$\therefore \cos^{-1}(2x-1)$ ৰ আদিক্ষেত্ৰ $[0, 1]$.

Q. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ আৰু $\cos^{-1}\left(-\frac{1}{2}\right)$ৰ মুখ্যমান কি.

Soln ইয়াত, $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\Rightarrow \cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$.

$\therefore x = \frac{\pi}{6}$.

আকৌ $\cos^{-1}\left(-\frac{1}{2}\right) = x$

$\Rightarrow \cos x = -\frac{1}{2} = \cos\frac{2\pi}{3}$

Q $\cos^{-1}\left[\sin\left(\cos^{-1}\frac{1}{2}\right)\right]$ ৰ মুখ্যমান উলিওৱা

Soln $\cos^{-1}\left[\sin\left(\cos^{-1}\frac{1}{2}\right)\right]$

$= \cos^{-1}\left[\sin\frac{\pi}{3}\right]$

$= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6}$. Ans

$\because \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Tangent ফলনৰ বিপৰীত ফলন: Inverse of Tangent f^n.

sin আৰু cos ফলনৰ দৰে আমি ইয়াত আদিক্ষেত্ৰ সীমিত কৰিব লাগিব। ই এটা বৰ্ধমান ফলন। θ ডাঙৰ হ'লে $\tan\theta$ ও ডাঙৰ হ'য়

$\tan: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ হ'লে ইয়াৰ বিপৰীত ফলন থাকিব।

$\tan^{-1}: R \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ আদিক্ষেত্ৰ বাস্তৱ সংখ্যা। সহক্ষেত্ৰ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(ইয়াত চন্দ্ৰবন্ধনী দিয়া হৈছে।

শর্ত৩, $-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$.

মনে রাখবে $\sin$, $\cos$ এর ক্ষেত্রে–
$-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$ আবশ্যিক

Q. মুখ্য মান নির্ণয় কর –
(i) $\tan^{-1}(-\sqrt{3})$ (ii) $\tan^{-1}(1)$ (iii) $\tan^{-1}\left\{\sin\left(-\frac{\pi}{2}\right)\right\}$ etc

Soln (i) Let $\tan^{-1}(-\sqrt{3}) = x$

$\Rightarrow \tan x = -\sqrt{3}$
$= -\tan\frac{\pi}{3}$
$= \tan\left(-\frac{\pi}{3}\right)$

$\therefore x = -\frac{\pi}{3}$ $\quad -\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(ii) $\tan^{-1}1 = x$
$\Rightarrow \tan x = 1 = \tan\frac{\pi}{4}$
$\therefore x = \frac{\pi}{4}$.

(iii) $\tan^{-1}\left\{\sin\left(-\frac{\pi}{2}\right)\right\}$
$= \tan^{-1}(-1)$
$= -\tan^{-1}(1)$
$= -\frac{\pi}{4}$

$\sin\left(-\frac{\pi}{2}\right)$
$= -\sin\frac{\pi}{2} = -1$

(iv) $\tan^{-1}\left\{\cos\frac{3\pi}{2}\right\}$
$= \tan^{-1}(0)$
$= 0$. Ans. //

$\cos\frac{3\pi}{2} = \cos\left(2\pi - \frac{\pi}{2}\right)$
$= \cos\frac{\pi}{2} = 0$

$\sin 30^\circ = \frac{1}{2}$
$\therefore \sin^{-1}\frac{1}{2} = 30^\circ = \frac{\pi}{6}$
$\cos 60^\circ = \frac{1}{2}$

$\tan\frac{\pi}{4} = 1$.

(v) $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$
$= \tan^{-1}\left\{2\cos\left(2\times\frac{\pi}{6}\right)\right\}$
$= \tan^{-1}\left\{2\cos\frac{\pi}{3}\right\}$
$= \tan^{-1}\left\{2\times\frac{1}{2}\right\}$
$= \tan^{-1}1$
$= \frac{\pi}{4}$

(i) $\cot[\sin^{-1}\{\cos(\tan^{-1}1)\}]$

$= \cot[\sin^{-1}\{\cos\frac{\pi}{4}\}]$

$= \cot(\sin^{-1}\frac{1}{\sqrt{2}})$

$= \cot\frac{\pi}{4}$

$= 1$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$.

$\tan\frac{\pi}{4} = \cot\frac{\pi}{4} = 1$

Ex কোনটো ডাঙৰ $\tan 1$ নে $\tan^{-1} 1$?

Ans. $\pi = \frac{22}{7}$ ধৰিলে; $\frac{\pi}{4} = \frac{22}{28}$.

$\therefore 1 > \frac{\pi}{4}$

$\Rightarrow \tan 1 > \tan\frac{\pi}{4}$ (যিহেতু tan এটা বৰ্ধমান ফলন)

$\Rightarrow \tan 1 > 1$

$\Rightarrow \tan 1 > 1 > \frac{\pi}{4}$.

$\Rightarrow \tan 1 > \frac{\pi}{4}$

$\Rightarrow \tan 1 > \tan^{-1} 1$

$\tan^{-1}1 = \frac{\pi}{4}$

$\tan\frac{\pi}{4} = 1$.

Inverse of ~~COTANT~~ Function. cotangent বা cot ফলনৰ বিপৰীত ফলন :-

আদিক্ষেত্ৰ $(0,\pi)$, সহক্ষেত্ৰ বাস্তৱ সংখ্যা R লেলে cotangent টো 1-1 আৰু onto হ'ব।

অৰ্থাৎ, $\cot:(0,\pi) \to R$ প্ৰতিলোমনীয়। ~~$\cot^{-1}$ হ'ব~~.

$\therefore \cot^{-1}: R \to (0,\pi)$ হ'ব।

মন কৰিবা cotangent টো এটা ক্ৰমহ্ৰাসমান (decreasing) ফলন।

Ex $\cot^{-1}1$ আৰু $\cot^{-1}(-1)$ ৰ মান উলিওৱা-

Let $x = \cot^{-1}1 \Rightarrow \cot x = 1 = \cot\frac{\pi}{4}$ $\therefore x = \frac{\pi}{4}$.

Let $y = \cot^{-1}(-1) \Rightarrow \cot y = -1 = -\cot\frac{\pi}{4} = \cot(\pi - \frac{\pi}{4})$

$= \cot\frac{3\pi}{4}$ $\therefore y = \frac{3\pi}{4}$

মন কৰা $-\cot\frac{\pi}{4} = \cot(-\frac{\pi}{4})$ লিখিব পৰা নাযায়। কাৰণ $0 < \theta < \pi$.

মুখ্য মান উলিওৱা:

(i) $\cot^{-1}\sqrt{3}$

ধৰা হ'ল $x = \cot^{-1}\sqrt{3}$

$\Rightarrow \cot x = \sqrt{3} = \cot\frac{\pi}{6}$

$\therefore x = \frac{\pi}{6}$. Ans.

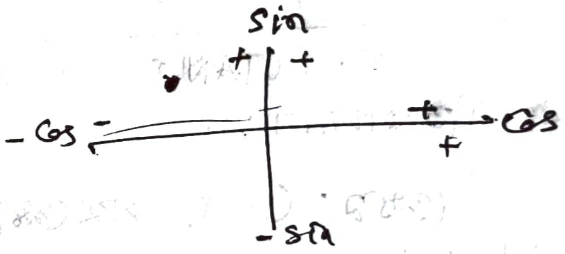
Inverse of secant function:-

তোমালোকে জানা যে $\sec\frac{\pi}{2} = \infty$ অৰ্থাৎ sec ফলনটো $-\frac{\pi}{2}, \frac{3\pi}{2}$ আদিত defined নহয় (নবহে)।

আমি যদি $[0, \pi]$ অন্তৰালৰ পৰা $\frac{\pi}{2}$ বাদ দিও তেন্তে
$\sec : [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi] \to (-\infty, -1] \cup [1, \infty)$ ফলনটো
1-1 আৰু আচ্ছাদক হয়।

অৰ্থাৎ $\sec^{-1} : (-\infty, -1] \cup [1, \infty) \to [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ হয়।

Ex. (i) $\sec^{-1}2$ (ii) $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$ ৰ মুখ্য মান (Principal value) উলিওৱা।



Sol^n (i) Let $x = \sec^{-1}2$

$\Rightarrow \sec x = 2$

$\Rightarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore x = \frac{\pi}{3}$. Ans [$\because \frac{\pi}{3} \in [0, \frac{\pi}{2})$ গতিকে ok]

(ii) Let $y = \sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

$\Rightarrow \sec y = -\frac{2}{\sqrt{3}}$

$\Rightarrow \cos y = -\frac{\sqrt{3}}{2} = \cancel{-\cos\pi}$

$= -\cos\frac{\pi}{6} = \cos\left(\pi - \frac{\pi}{6}\right) = \cos\frac{5\pi}{6}$

$\therefore y = \frac{5\pi}{6}$ Answer.

(iii) $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ ৰ মুখ্যমান উলিওৱা

Sol^n sec ফলনটো ~~⟶~~ $x \le -1$, বা $x \ge 1$ ৰ ক্ষেত্ৰত হে প্ৰযোজ্য। $\frac{\sqrt{3}}{2} < 1$ $\therefore \sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ অৰ্থহীন।

অসংজ্ঞায়িত বিধায় আমরা বলতে পারি যে $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ এর মান অসংজ্ঞায়িত ϕ.

Ex. (i) $\sec^{-1}\frac{2}{\sqrt{3}}$ (ii) $\sec^{-1}(-2)$ নির্ণয় কর।

Let $x = \sec^{-1}\frac{2}{\sqrt{3}}$

$\Rightarrow \sec x = \frac{2}{\sqrt{3}} \Rightarrow \cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$ $\therefore x = \frac{\pi}{6}$.

(ii) $y = \sec^{-1}(-2) \Rightarrow \sec y = -2 \Rightarrow \cos y = -\frac{1}{2}$

$= -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$.

$\therefore$ $y = \frac{2\pi}{3}$ Ans.

Q. $\sec^{-1}(2x+1)$ এর ডোমেন নির্ণয় কর:

Soln. Domain হলো $(-\infty, -1] \cup [1, \infty)$

$\therefore$ $\sec^{-1}(2x+1)$ অর্থপূর্ণ হবে যদি

$2x+1 \le -1$ বা $1 \le 2x+1$

$\Rightarrow$ $2x \le -2$ বা $0 \le 2x$

$\Rightarrow$ $x \le -1$ বা $0 \le x$

$\Rightarrow$ $x \in (-\infty, -1] \cup [0, \infty)$.

$\therefore$ $\sec^{-1}(2x+1)$ এর ডোমেন $(-\infty, -1] \cup [0, \infty)$. Ans.