$\therefore x = \frac{\pi}{6}$. Ans .

Inverse of secant function:—

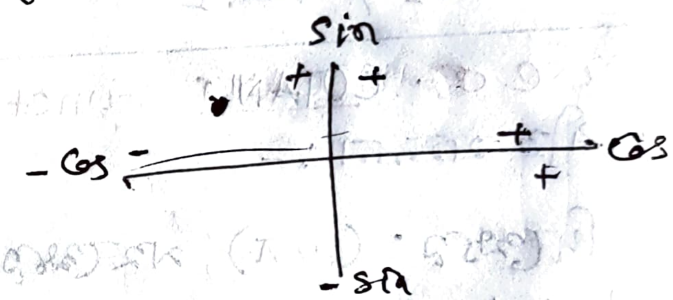
তোমালোকে জানা যে $\sec\frac{\pi}{2} = \infty$ অৰ্থাৎ sec ফাংচনটো $\frac{\pi}{2}, \frac{3\pi}{2}$ আদিত defined নহয় (নবৰ্তে)।

আমি যদি $[0, \pi]$ অন্তৰালৰ পৰা $\frac{\pi}{2}$ বাদ দিওঁ তেন্তে

$\sec : [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi] \to (-\infty, -1] \cup [1, \infty)$ ফাংচনটো 1-1 আৰু আচ্ছাদক হ'ব।

অৰ্থাৎ $\sec^{-1} : (-\infty, -1] \cup [1, \infty) \to [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ হ'ব।

Ex. (i) $\sec^{-1}2$ (ii) $\sec^{-1}(-\frac{2}{\sqrt{3}})$ ৰ মুখ্য মান (Principal value) উলিওৱা।



Solⁿ (i) Let $x = \sec^{-1}2$

$\Rightarrow \sec x = 2$

$\Rightarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore x = \frac{\pi}{3}$. Ans $[\because \frac{\pi}{3} \in [0, \frac{\pi}{2})$ গতিকে ok$]$

(ii) Let $y = \sec^{-1}(-\frac{2}{\sqrt{3}})$

$\Rightarrow \sec y = -\frac{2}{\sqrt{3}}$

$\Rightarrow \cos y = -\frac{\sqrt{3}}{2} = \cancel{-\cos\pi}$

$= -\cos\frac{\pi}{6} = \cos(\pi - \frac{\pi}{6}) = \cos\frac{5\pi}{6}$

$\therefore y = \frac{5\pi}{6}$ Answer.

(iii) $\sec^{-1}(\frac{\sqrt{3}}{2})$ ৰ মুখ্যমান উলিওৱা

Solⁿ sec ফাংচনটো $x \le -1$, বা $x \ge 1$ ৰ ক্ষেত্ৰত হে প্ৰযোজ্য। $\frac{\sqrt{3}}{2} < 1$ $\therefore \sec^{-1}(\frac{\sqrt{3}}{2})$ অৰ্থহীন।

অনুরূপ ভাবে আমরা বলতে পারি যে $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ এর মান অসংজ্ঞায়িত ϕ.

Ex. (i) $\sec^{-1}\frac{2}{\sqrt{3}}$ (ii) $\sec^{-1}(-2)$ নির্ণয় কর।

Let $x = \sec^{-1}\frac{2}{\sqrt{3}}$

$\Rightarrow \sec x = \frac{2}{\sqrt{3}} \Rightarrow \cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \quad \therefore x = \frac{\pi}{6}$.

(ii) $y = \sec^{-1}(-2) \Rightarrow \sec y = -2 \Rightarrow \cos y = -\frac{1}{2}$

$= -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$.

$\therefore \quad y = \frac{2\pi}{3}$ Ans.

Q. $\sec^{-1}(2x+1)$ এর আধিক্ষেত্র নির্ণয় কর:

Soln. Domain হল $(-\infty, -1] \cup [1, \infty)$

$\therefore$ $\sec^{-1}(2x+1)$ অর্থপূর্ণ হবে যদি ~~$x \geq -1$ বা $x \geq 1$~~

$2x+1 \leq -1$ বা $1 \leq 2x+1$

$\Rightarrow \quad 2x \leq -2$ বা $0 \leq 2x$.

$\Rightarrow \quad x \leq -1$ বা $0 \leq x$

$\Rightarrow \quad x \in (-\infty, -1] \cup [0, \infty)$ ~~Ans~~.

$\therefore$ $\sec^{-1}(2x+1)$ এর আধিক্ষেত্র $(-\infty, -1] \cup [0, \infty)$. Ans

~~Example~~ INVERSE OF COSECANT FUNCTION.

(cosec ফলনৰ বিপৰীত ফলন)

cosec ফলনটো f ললে $f : [-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}] \to (-\infty, -1] \cup [1, \infty)$

ফলনটো একৈকী আৰু আচ্ছাদক। গতিকে এই Domain আৰু codomain ত $cosec^{-1}$ বৰ্তে।

গতিকে $cosec^{-1}$ ক আমি তলত দিয়া ধৰণে সংজ্ঞাবদ্ধ কৰিব পাৰো।

$$cosec^{-1}x = \theta \Leftrightarrow cosec\,\theta = x \ \forall\, \theta \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$$

আৰু $x \in (-\infty, -1] \cup [1, \infty)$.

Q1 তলৰ cosec^{-1} ফলনৰ কেইটাৰ মুখ্যমান উলিওৱা।

(i) $\text{cosec}^{-1}(-\sqrt{2})$

ধৰা $\text{cosec}^{-1}(-\sqrt{2}) = x$

$\Rightarrow \text{cosec}\, x = -\sqrt{2}$

$\Rightarrow \sin x = -\frac{1}{\sqrt{2}}$

$= -\sin\left(\frac{\pi}{4}\right)$

$= \sin\left(-\frac{\pi}{4}\right)$

$\therefore x = -\frac{\pi}{4}$

মন কৰা,
যিহেতু $-\frac{\pi}{4} \in \left[-\frac{\pi}{2}, 0\right)$
গতিকে আমি x ৰ মান $-\frac{\pi}{4}$ ল'ব পাৰো।

(ii) $\text{cosec}^{-1}(2)$.

ধৰা $\text{cosec}^{-1} 2 = x$

$\Rightarrow \text{cosec}\, x = 2$

$\Rightarrow \sin x = \frac{1}{2}$

$= \sin \frac{\pi}{6}$

$\therefore x \in \frac{\pi}{6}$.

(iii) $\text{cosec}^{-1}(-2)$

$\Rightarrow \text{cosec}\, x = -2$

$\Rightarrow \sin x = -\frac{1}{2}$

$= -\sin\left(\frac{\pi}{6}\right)$

$= \sin\left(-\frac{\pi}{6}\right)$

$\therefore x = -\frac{\pi}{6}$.

(iv) $\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

Let $\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right) = x$

$\Rightarrow \operatorname{cosec} x = -\frac{2}{\sqrt{3}}$

$\Rightarrow \sin x = -\frac{\sqrt{3}}{2}$

$= -\sin\frac{\pi}{3}$

$= \sin\left(-\frac{\pi}{3}\right)$

$\therefore x = -\frac{\pi}{3}$.

(v) $\operatorname{cosec}^{-1}\left(2\cos\frac{2\pi}{3}\right)$

$= \operatorname{cosec}^{-1}\left[2\times\cos\left(\pi-\frac{\pi}{3}\right)\right]$

$= \operatorname{cosec}^{-1}\left[2\times\cos\frac{\pi}{3}\right]$

$= \operatorname{cosec}^{-1}\left[2\times\frac{1}{2}\right]$

$= \operatorname{cosec}^{-1} 1$

ধৰা হওঁক $\operatorname{cosec}^{-1} 1 = x$

$\Rightarrow \operatorname{cosec} x = 1$

$\Rightarrow \sin x = 1 = \sin\frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{2}$

$\frac{\pi}{2}$

π — 0

(v) Let $\operatorname{cosec}^{-1}\left(2\cos\frac{2\pi}{3}\right) = x$

$\Rightarrow \operatorname{cosec} x = 2\cos\frac{2\pi}{3}$

$= 2\times\cos\left(\pi-\frac{\pi}{3}\right)$

(দ্বিতীয় ক'ৰত $\cos(\pi-\theta) = -\cos\theta$)

$= -2\cos\frac{\pi}{3}$

$= -2\times\frac{1}{2}$

$= -1$ ক।

$\Rightarrow \sin x = -1$

$= -\sin\left(\frac{\pi}{2}\right)$

$= \sin\left(-\frac{\pi}{2}\right)$

$\therefore x = -\frac{\pi}{2}$. Ans

মান নিৰ্ণয় কৰা :-

(i) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) + \csc^{-1}\frac{2}{\sqrt{3}}$

Soln $\frac{\pi}{3} - \frac{2\pi}{3} + \frac{\pi}{3} = 0$. Ans.

$\tan^{-1}\sqrt{3} = x$

$\tan x = \sqrt{3}$

$= \tan\frac{\pi}{3}$

$\therefore x = \frac{\pi}{3}$ etc.

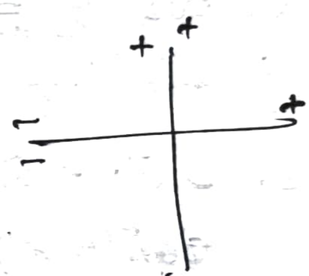
(ii) $2\sec^{-1}(2) - 2\csc^{-1}(-2)$

$= 2\times\frac{\pi}{3} - 2\times\left(-\frac{\pi}{6}\right)$

$= \pi$. Ans

Q $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ ৰ মান নিৰ্ণয় কৰা.

$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) \neq \frac{7\pi}{6}$ কাৰণ $\frac{7\pi}{6} \notin [0, \pi]$



Now, $\cos^{-1}\left[\cos\frac{7\pi}{6}\right] = \cos^{-1}\left[\cos\left(\pi + \frac{\pi}{6}\right)\right]$

$= \cos^{-1}\left[-\cos\frac{\pi}{6}\right]$ (অল-কো-সমস্যা)

$= \cos^{-1}\left[\cos\left(\pi - \frac{5\pi}{6}\right)\right]$

$= \cos^{-1}\left(\cos\frac{5\pi}{6}\right)$

$= \frac{5\pi}{6}$. Ans.

$\frac{\pi}{2}$

(vii) $\sin^{-1}(\sin(-600^\circ))$

$= \sin^{-1}\left\{\sin\left(-600\times\frac{\pi}{180}\right)\right\}$ (note)

$= \sin^{-1}\left\{\sin\left(-\frac{10\pi}{3}\right)\right\}$

$= \sin^{-1}\left\{-\sin\frac{10\pi}{3}\right\}$ $(\because \sin(-\theta) = -\sin\theta)$

$= \sin^{-1}\left\{-\sin\left(3\pi + \frac{\pi}{3}\right)\right\}$

$= \sin^{-1}\left\{-\left(-\sin\frac{\pi}{3}\right)\right\}$

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$= \frac{\pi}{3}$ Answer.

(viii) $\cos^{-1}\{\cos(-680^\circ)\}$

$= \cos^{-1}(\cos 680^\circ)$ $\quad [\because \cos(-\theta) = \cos\theta]$

$= \cos^{-1}\left\{\cos 680 \times \frac{\pi}{180}\right\}$

$= \cos^{-1}\left\{\cos \frac{34\pi}{9}\right\}$

$= \cos^{-1}\left\{\cos\left(4\pi - \frac{2\pi}{9}\right)\right\}$

$= \cos^{-1}\left\{\cos \frac{2\pi}{9}\right\}$

$= \frac{2\pi}{9}$ Ans

চতুৰ্থ চোকত $(4\pi - \theta)$

Properties of Inverse trigonometric functions:-

1.
(i) $\sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}x$
(ii) $\cos^{-1}\frac{1}{x} = \sec^{-1}x$.
(iii) $\tan^{-1}\frac{1}{x} = \cot^{-1}x$.

2. $\sin^{-1}(-x) = -\sin^{-1}x$
$\tan^{-1}(-x) = -\tan^{-1}x$
$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$

3. $\cos^{-1}(-x) = \pi - \operatorname{cosec}^{-1}x$
$\cot^{-1}(-x) = \pi - \cot^{-1}x$
$\sec^{-1}(-x) = \pi - \sec^{-1}x$.

4. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$
$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$
$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

5. (i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

Proof: Let $\tan^{-1}x = \alpha$. $\Rightarrow \tan\alpha = x$

$\tan^{-1}y = \beta \Rightarrow \tan\beta = y$

Now, $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$ (ত্রিকোণমিতিক নিয়ম)

$$= \frac{x+y}{1-xy}$$

$$\Rightarrow \alpha+\beta = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$\Rightarrow \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ Proved.}$$

একইভাবে-

(ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$

6.(i) $2\tan^{-1}x = \sin^{-1}\dfrac{2x}{1+x^2}$

Let $\tan^{-1}x = \theta \Rightarrow x = \tan\theta$.

$\therefore$ Now $\sin^{-1}\dfrac{2\tan\theta}{1+\tan^2\theta}$

$$= \sin^{-1}\frac{2\tan\theta}{\sec^2\theta}$$

$$= \sin^{-1}\left\{2\times\frac{\sin\theta}{\cos\theta}\times\cos^2\theta\right\}$$

$$= \sin^{-1}\{2\sin\theta\cos\theta\}$$

$$= \sin^{-1}\sin 2\theta$$

$$= 2\theta$$

$1+\tan^2\theta = \sec^2\theta$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$2\sin\theta\cos\theta = \sin 2\theta$

Thus $2\theta = \sin^{-1}\dfrac{\tan\theta}{1+\tan^2\theta}$

$$\Rightarrow 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}. \text{ Proved}$$

(ii) ~~$\cos^{-1}\frac{1-x^2}{\ldots}$~~ $= 2\tan^{-1}x$

(ii) $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2},\ x \geq 0.$

ধরো $\tan^{-1}x = \theta \Rightarrow \tan\theta = x$

এখন $\cos^{-1}\frac{1-x^2}{1+x^2} = \cos^{-1}.\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \cos^{-1}.\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta.\sec^2\theta}$

$= \cos^{-1}(\cos^2\theta - \sin^2\theta)$

$= \cos^{-1}(\cos 2\theta)$

$= 2\theta.$

$= 2\tan^{-1}x.$

Thus $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}$

$1-\tan^2\theta$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}$

$1+\tan^2\theta = \sec^2\theta$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$\cos^2\theta.\sec^2\theta = 1$

(iii) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

$\tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{2\tan\theta}{1-\tan^2\theta}$

$= \cancel{\tan^{-1}}$

$= \tan^{-1}\frac{2\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}$

$= \tan^{-1}\frac{2.\frac{\sin\theta}{\cos\theta}.\cos^2\theta}{\cos^2\theta - \sin^2\theta}$

$= \tan^{-1}\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

$= \tan^{-1}\frac{\sin 2\theta}{\cos 2\theta}$

$= \tan^{-1}(\tan 2\theta)$

$= 2\theta$

$= 2\tan^{-1}x.$ Ans.

$2x\ \frac{\sin\theta}{\cos\theta}$

$1-\tan^2\theta = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}$

$2\frac{\sin\theta}{\cos\theta} \times \frac{\cos^2\theta}{\cos 2\theta}$

Example 3 Show that

① $\sin(2x\sqrt{1-x^2}) = 2\sin^{-1}x \quad \left(\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}\right)$

Sol^n ধরা $x = \sin\theta$. তাহলে $\theta = \sin^{-1}x$

Now, $\sin^{-1}(2x\sqrt{1-x^2}) = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta$.

$= 2\sin^{-1}x$ Proved.

$\boxed{1-\sin^2\theta = \cos^2\theta}$

Q. $\cot(\tan^{-1}a + \cot^{-1}a)$ এর মান নির্ণয় করো।

Sol^n $\cot(\tan^{-1}a + \cot^{-1}a) = \cot\frac{\pi}{2} = 0$.

$\cot^{-1}\theta + \tan^{-1}\theta = \frac{\pi}{2}$.

Q. $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$ হ'লে

$\cos^{-1}x + \cos^{-1}y = ?$

Sol^n $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} - \sin^{-1}x + \frac{\pi}{2} - \sin^{-1}y = \frac{\pi}{2}$

$\Rightarrow \cancel{\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}}$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$ //

Q. $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$ হ'লে $\cot^{-1}x + \cot^{-1}y = ?$

Sol^n $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{2} - \cot^{-1}x + \frac{\pi}{2} - \cot^{-1}y = \frac{\pi}{2} + \frac{\pi}{2} - \frac{4\pi}{5}$

Sol^n. $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$ (Given)

$\Rightarrow \frac{\pi}{2} - \tan^{-1}x + \frac{\pi}{2} - \tan^{-1}y = \frac{\pi}{2} + \frac{\pi}{2} - \frac{4\pi}{5}$

$\Rightarrow \cot^{-1}x + \cot^{-1}y = \frac{\pi}{5}$ Ans

সমাধান কর:

8. ~~[illegible]~~ $\quad$ (i) $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

(i) ~~$\tan^{-1}$ [illegible] $\tan^{-1} 8$~~ $\quad$ (ii) $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

(iii) $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{36}$.

Sol^n (i) $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \tan^{-1}1$

$\Rightarrow \tan^{-1}\frac{x-1}{x-2} = \tan^{-1}1 - \tan^{-1}\frac{x+1}{x+2}$

$= \tan^{-1}\dfrac{1-\frac{x+1}{x+2}}{1+1\cdot\frac{x+1}{x+2}}$

$= \tan^{-1}\dfrac{x+2-x-1}{x+2+x+1}$

$= \tan^{-1}\dfrac{1}{2x+3}$

$\Rightarrow \frac{x-1}{x-2} = \frac{1}{2x+3}$

$\Rightarrow 2x^2+x-3 = x-2$

$\Rightarrow 2x^2-1=0 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

(ii) $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left\{\dfrac{2x+3x}{1-2x\times 3x}\right\} = \tan^{-1}1$ $\quad$ (যেখানে $6x^2<1$)

$\Rightarrow \frac{5x}{1-6x^2} = 1$ এবং $x^2 < \frac{1}{6}$.

$\Rightarrow 6x^2+5x-1=0$ এবং $-\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$.

$\Rightarrow$ ~~$x=$~~ $(6x-1)(x+1)=0$ and $-\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$

$\Rightarrow x = -1, \frac{1}{6}$ এবং $-\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$

$\Rightarrow x = \frac{1}{6}$ Ans.

Q. যদি $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\frac{1}{\sqrt{3}}$, x এর মান নির্ণয় করো।

Solⁿ $\tan^{-1}x - \cot^{-1}x = \tan^{-1}1/\sqrt{3}$

$\Rightarrow \tan^{-1}x - \cot^{-1}x = \frac{\pi}{6}$ ——— (1)

we know that $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ ——— (2)

$\frac{4\pi}{6} = \frac{2}{3}$

(1)+(2) $\Rightarrow 2\tan^{-1}x = \frac{\pi}{6} + \frac{\pi}{2}$

$\Rightarrow \tan^{-1}x = \frac{\pi}{3}$

$= \tan^{-1}\sqrt{3}$

$\therefore x = \sqrt{3}$. Ans.

Ex. যদি $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, find x

Solⁿ $\sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}1$

$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}\frac{1}{5}$

$\Rightarrow \cos^{-1}x = \cos^{-1}\frac{1}{5}$. $\left(\because \sin^{-1}\frac{1}{5} + \cos^{-1}\frac{1}{5} = \frac{\pi}{2}\right)$

$\Rightarrow x = 1/5$ Ans.